23 "End behavior of a polynomial matches the end behav.. of its highest degree term"

Limit-speak:

$$\lim_{x \to \overset{(-)}{\infty}} c_n x^n + c_{n-1}x^{n-1} + \dots + c_1 x^1 + c_0 = \lim_{x \to \overset{(-)}{\infty}} c_n x^n$$

$$\lim_{x \to -\infty} 7x^5 - 4x^3 + 2x - 9$$

$$= \lim_{x \to -\infty} 7x^5 = -\infty$$

$$\lim_{x \to -\infty} -4x^8 + 17x^3 - 5x + 1 = \lim_{x \to -\infty} -4x^8 = -\infty$$

$\lim_{x \to \infty} c_n x^n = +\infty$ if n even, $-\infty$ if n odd

[assuming $c_n > 0$]

$\lim_{x \to \infty} c_n x^n$: $+\infty$ if $c_n > 0$, $-\infty$ if $c_n < 0$

$$\lim_{x\to+\infty} \frac{3x+5}{6x-8} \qquad x\left(\frac{3x}{x} + \frac{5}{x}\right)$$

$$= \lim_{x\to+\infty} \frac{x\left(3+\frac{5}{x}\right)}{x\left(6-\frac{8}{x}\right)}$$

$$\lim_{x\to+\infty} \frac{3x}{6x} = \frac{3}{6}$$

$$= \lim_{x\to+\infty} \frac{3+\frac{5}{x}}{6-\frac{8}{x}} = \frac{1}{2}$$

$$\lim_{x\to\infty} \frac{5}{x} =$$

x	$\frac{5}{x}$
10	$\frac{5}{10}$
1000	$\frac{5}{1000}$
10^8	$\frac{5}{10^8}$

$$\lim_{x\to-\infty} \frac{4x^2 - x}{2x^3 - 5}$$

$$\lim_{x\to-\infty} \frac{x^2\left(4 - \frac{1}{x}\right)}{x^3\left(2 - \frac{5}{x^3}\right)}$$

$$= \lim_{x\to-\infty} \frac{4 - \frac{1}{x}}{x\left(2 - \frac{5}{x^3}\right)}$$

$$\lim_{x\to-\infty} \frac{4x^2}{2x^3}$$

$$= 0$$

$$\lim_{x\to-\infty} \frac{5x^3 - 2x^2 + 1}{3x + 5}$$

$$= \lim_{x\to-\infty} \frac{x^3\left(5 - \frac{2}{x} + \frac{1}{x^3}\right)}{x\left(3 + \frac{5}{x}\right)}$$

$$= \lim_{x\to-\infty} \frac{x^2\left(5 - \frac{2}{x} + \frac{1}{x^3}\right)}{\left(3 + \frac{5}{x}\right)}$$

$$= +\infty$$

$$\lim_{x \to -\infty} \frac{4x^2 - x}{2x^3 - 5}$$

$$\lim_{x \to -\infty} \frac{x^3\left(\frac{4x^2}{x^3} - \frac{x}{x^3}\right)}{x^3\left(2 - \frac{5}{x^3}\right)}$$

$$= \lim_{x \to -\infty} \frac{\frac{4}{x} - \frac{1}{x^2}}{\left(2 - \frac{5}{x^3}\right)} \quad \frac{0}{2}$$

$$\lim_{x \to -\infty} \frac{x^2\left(4 - \frac{x}{x^2}\right)}{x^2\left(2x - \frac{5}{x^2}\right)}$$

$$\lim_{x \to -\infty} \frac{4 - \frac{1}{x}}{2x - \frac{5}{x^2}} \quad \begin{matrix} \to 4 \\ \to -\infty \end{matrix}$$

$$= 0$$

$$-\frac{4}{\infty}$$

$$\lim_{x\to+\infty} \sqrt[3]{\frac{3x+5}{6x-8}} = \sqrt[3]{\frac{1}{2}}$$

$$\lim_{x\to+\infty} \sqrt[3]{\frac{x\left(3+\frac{5}{x}\right)}{x\left(6-\frac{8}{x}\right)}}$$

$$\lim_{x\to+\infty} \sqrt[3]{\frac{3+\frac{5}{x}}{6-\frac{8}{x}}}$$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$\lim_{x\to+\infty} \frac{\sqrt{x^2\left(1+\frac{2}{x^2}\right)}}{x\left(3-\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\overset{x}{\sqrt{x^2}}\,\sqrt{1+\frac{2}{x^2}}}{x\left(3-\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}} = \frac{\sqrt{1}}{3} = \frac{1}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6} \quad \frac{+}{-} = -$$

$$\sqrt{(2)^2} = 2$$

$$\sqrt{(-2)^2} = 2$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{2}{x^2}}}{x\left(3-\frac{6}{x}\right)} \quad \frac{+}{-}$$

$$\frac{\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)} = +$$

$$= \lim_{x\to-\infty} \frac{|x|\sqrt{1+\frac{2}{x^2}}}{x\left(3-\frac{6}{x}\right)} \quad \frac{-}{-} = +$$

$$= \lim_{x\to-\infty} -\left(\frac{\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}}\right) = -\frac{1}{3}$$

$$\lim_{x\to+\infty}\left(\sqrt{x^6+5}-x^3\right)$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^6+5}-x^3\right)}{1}\cdot\frac{\left(\sqrt{x^6+5}+x^3\right)}{\left(\sqrt{x^6+5}+x^3\right)}$$

$$=\lim_{x\to+\infty}\frac{(x^6+5)-x^6}{\sqrt{x^6+5}+x^3}=\lim_{x\to\infty}\frac{5}{\sqrt{x^6+5}+x^3}$$

$$=0$$

$$\lim_{x\to+\infty}\left(\sqrt{x^6+5}-x^4\right)$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^6+5}-x^4\right)}{1}\cdot\frac{\left(\sqrt{x^6+5}+x^4\right)}{\left(\sqrt{x^6+5}+x^4\right)}$$

$$=\lim_{x\to\infty}\frac{x^6+5-x^8}{\sqrt{x^6+5}+x^4}$$

$$=\lim_{x\to\infty}\frac{-x^8+x^6+5}{\sqrt{x^6+5}+x^4}$$

$$\sqrt{x^8}\sqrt{\frac{1}{x^2}+\frac{5}{x^8}}$$

$$=\lim_{x\to\infty}\frac{x^8\left(-1+\frac{1}{x^2}+\frac{5}{x^8}\right)}{x^4\left(\sqrt{\frac{1}{x^2}+\frac{5}{x^8}}+1\right)}$$

$$=\lim_{x\to\infty}\frac{x^4\left(-1+\frac{1}{x^2}+\frac{5}{x^8}\right)}{\sqrt{1/x^2+5/x^8}+1}$$

$$=\lim_{x\to\infty}\frac{x^4(-1)}{1}=-\infty$$

$$\lim_{x\to\infty} \sqrt{x^6+5x^3} - x^3$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^6+5x^3} - x^3}{1}\right)\left(\frac{\sqrt{x^6+5x^3}+x^3}{\sqrt{x^6+5x^3}+x^3}\right)$$

$$= \lim_{x\to\infty} \frac{(x^6+5x^3)-x^6}{\sqrt{x^6+5x^3}+x^3}$$

$$= \lim_{x\to\infty} \frac{5x^3}{\sqrt{x^6+5x^3}+x^3}$$

$$\lim_{x\to\infty} \frac{x^3(5)}{\sqrt{x^6}\left(\sqrt{1+\frac{5}{x^3}}\right)+x^3}$$

$$\boxed{x^3\sqrt{1+\frac{5}{x^3}}+x^3}$$

$$x^3\left(\sqrt{1+\frac{5}{x^3}}+1\right)$$

$$= \lim_{x\to\infty} \frac{x^3(5)}{x^3\left[\sqrt{1+\frac{5}{x^3}}+1\right]}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x^3}}+1}$$

$$= \frac{5}{1+1} = 2.5$$

$$\lim_{x\to\infty} \sqrt{x^6+5x^3} - x^3$$

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^6+5x^3} - x^3\right)}{1} \cdot \left(\frac{\sqrt{x^6+5x^3}+x^3}{\sqrt{x^6+5x^3}+x^3}\right)$$

$$\lim_{x\to\infty} \frac{(x^6+5x^3) - x^6}{\sqrt{x^6+5x^3} + x^3}$$

$$\lim_{x\to\infty} \frac{5x^3}{\sqrt{x^6+5x^3}+x^3}$$

$$\lim_{x\to\infty} \frac{x^3(5)}{x^3\sqrt{1+\frac{5}{x^3}} + x^3}$$

$$= \lim_{x\to\infty} \frac{x^3(5)}{x^3\left(\sqrt{1+\frac{5}{x^3}} + 1\right)}$$

$$= \lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x^3}} + 1} = \frac{5}{\sqrt{1}+1}$$

$$= 2.5$$

$$\lim_{x\to\infty} \sqrt{x^6+5x^3} - x^3$$

$\infty - \infty$ STOP

$$\lim_{x\to\infty} \sqrt{x^6}\sqrt{1+\frac{5}{x^3}} - x^3$$

$$= \lim_{x\to\infty} x^3\left(\sqrt{1+\frac{5}{x^3}} - 1\right)$$

$\infty(0)$?

Indeterminate form